Hong Kong Mathematics Olympiad (2018/19)

Finals (Group – Event 1)

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time	Min. / Sec.
			Total score					

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. Let $x+y=32$ with $x, y \geq 0$. If a is the maximum value of $\sqrt{x}+\sqrt{y}$, determine the value of a.
已知 $x+y=32$，其中 $x, y \geq 0$。若 a 為 $\sqrt{x}+\sqrt{y}$ 的最大值，求 a 的值。

$a=$

2. A box contains only x one-dollar coins, $x+2$ two-dollar coins and $x+4$ five-dollar coins. If the probability of drawing a one-dollar coin randomly from the box is less than 0.1. If the box contains b coins, determine the value of b.
一個盒中只有 x 個一元硬幣，$x+2$ 個二元硬幣及 $x+4$ 個五元硬幣。隨機從盒中拿出一元硬幣的概率小於 0.1。若盒中有 b 個硬幣，求 b 的值。

$b=$

3. If c is the greatest common factor of the following numbers

$$3^3-3,\ 5^5-5,\ 7^7-7,\ 9^9-9,\ \ldots,\ 2019^{2019}-2019,$$

determine the value of c.
若 c 是以下數的最大公因數，

$$3^3-3,\ 5^5-5,\ 7^7-7,\ 9^9-9,\ \ldots,\ 2019^{2019}-2019,$$

求 c 的值。

$c=$

4. Let $x=\frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}-\sqrt{7}}$ and $y=\frac{\sqrt{5}-\sqrt{7}}{\sqrt{5}+\sqrt{7}}$. If $d=3x^2-7xy+3y^2$, determine the value of d.

設 $x=\frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}-\sqrt{7}}$ 和 $y=\frac{\sqrt{5}-\sqrt{7}}{\sqrt{5}+\sqrt{7}}$。若 $d=3x^2-7xy+3y^2$，求 d 的值。

$d=$

Hong Kong Mathematics Olympiad (2018/19)

Finals (Group – Event 2)

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			+ Bonus score				Time	
			Total score				Min.	Sec.

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.
除非特別聲明，答案須用數字表達，並化至最簡。

1. Let

$$X=\sqrt{2020-\sqrt{A}}$$

be a positive integer. Determine the least value of A.
設

$$X=\sqrt{2020-\sqrt{A}}$$

為正整數，求 A 的最小值。

$A=$

2. Suppose that

$$\begin{cases} x+y=5 \\ 4x^2+y^2=80 \end{cases}$$

and $P=(x_1, y_1)$ and $Q=(x_2, y_2)$ simultaneously satisfy these two equations. If $B=y_1-x_1+y_2-x_2$, determine the value of B.
假設

$$\begin{cases} x+y=5 \\ 4x^2+y^2=80 \end{cases}$$

及 $P=(x_1, y_1)$ 和 $Q=(x_2, y_2)$ 同時滿足這兩個等式。若 $B=y_1-x_1+y_2-x_2$，求 B 的值。

$B=$

3. If $Q=a^b-b^a$ is a positive integer, determine the least value of Q.

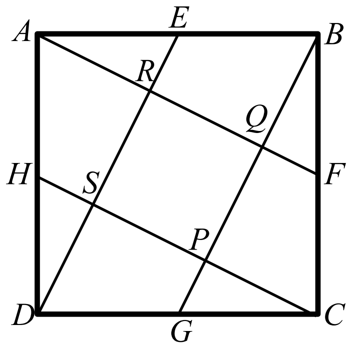
若 $Q=a^b-b^a$ 為正整數，求 Q 的最小值。

$n=$

4. In square $ABCD$, E, F, G and H are the mid-points of AB, BC, CD and AD respectively. DE intersects with AF and CH at R and S respectively. Moreover, BG intersects with AF and CH at Q and P respectively. If U is the area of square $ABCD$

and V is the area of quadrilateral $PQRS$, determine the value of $W=\frac{U}{V}$.

在正方形 $ABCD$ 中，E, F, G 和 H 分別是 AB, BC, CD 和 AD 的中點。DE 分別與線段 AF 和 CH 相交於點 R 和 S。BG 分別與線段 AF 和 CH 相交於點 Q 和 P。若 U 是正方形 $ABCD$ 的面積，而 V 是四邊形 $PQRS$ 的面積，求 $W=\frac{U}{V}$ 的值。



$W=$

Hong Kong Mathematics Olympiad (2018/19)

Finals (Group – Event 3)

FOR OFFICIAL USE

Score for accuracy		×	Mult. factor for speed		=		Team No.	
			+ Bonus score				Time		
			Total score					Min.	Sec.

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除非特別聲明，答案須用數字表達，並化至最簡。

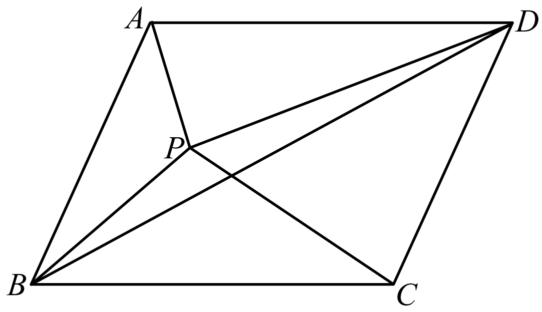
1. If $\sqrt{32\times81\times343}=b\sqrt{a}$, where a and b are positive integers, determine the least value of a.

若 $\sqrt{32\times81\times343}=b\sqrt{a}$，其中 a 和 b 是正整數，求 a 的最小值。

$a=$

2. In the diagram below, point P is inside parallelogram $ABCD$. If the areas of ΔABP, ΔBPC and ΔBPD are 73 cm^2, 100 cm^2, and e cm^2 respectively, determine the value of e.

下圖中，P 點在平行四邊形 $ABCD$ 內。若 ΔABP, ΔBPC 和 ΔBPD 的面積分別為 73 cm^2 、 100 cm^2 和 e cm^2，求 e 的值。



$e=$

3. A 3×3 magic square is filled with a number in each square such that the sum of the three numbers in each row, column and the two main diagonals are equal. The partially completed grid is shown below. Determine the value of c .

在以下的 3×3 幻方中，每行、列和兩斜行〔對角線〕的和相等。如下圖所示，部份數值已經填上。求 c 的值。

c	16	20
2		

$c =$

4. If $X = 2^{2018} + 3^{2018}$ and d is its unit digit, determine the value of d .

若 $X = 2^{2018} + 3^{2018}$ 及 d 是其個位數，求 d 的值。

$d =$

Hong Kong Mathematics Olympiad (2018/19)

Finals (Group – Event 4)

FOR OFFICIAL USE

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			Total score					

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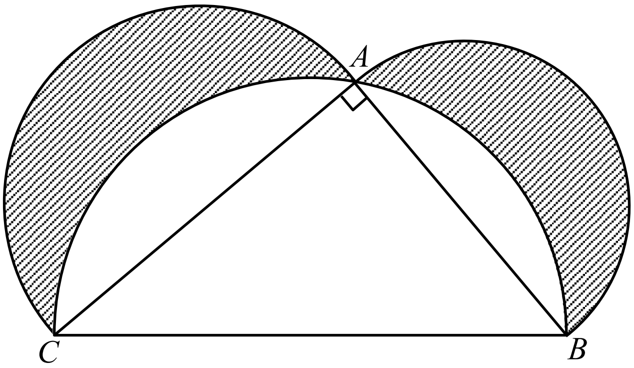
除非特別聲明，答案須用數字表達，並化至最簡。

1. ΔABC is a right-angled triangle with AC = 8 and BC = 10. Semi-circles are drawn with AB , AC and BC as diameters as shown. If the total shaded area is α, determine the value of α.
ΔABC 是直角三角形，AC = 8，BC = 10。以 AB、AC 和 BC 為直徑分別畫了三個半圓，如圖所示。若陰影部分的總面積是 α，求 α 的值。



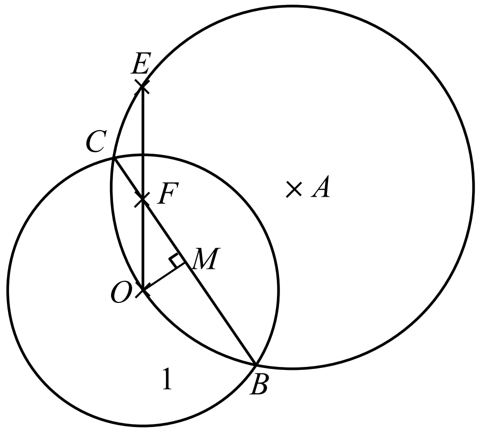
$\alpha =$

2. For all positive integers n, suppose there exists a function $F(n)$ defined for as follows:
$F(1)=0$,
for all $n \geq 2$,
$F(n)=F(n-1)+2$ if 2 divides n but 3 does not divide n;
$F(n)=F(n-1)+3$ if 3 divides n but 2 does not divide n;
$F(n)=F(n-1)+4$ if 2 and 3 both divide n ;
$F(n)=F(n-1)$ if neither 2 nor 3 divides n .
If $\beta=F(4000)$, determine the value of β.
對所有的正整數 n，設某一個函數 $F(n)$ 有如下定義：
$F(1)=0$ ，
當 $n \geq 2$，
如果 n 只能被2整除而不能被3整除，則 $F(n)=F(n-1)+2$ ；
如果 n 只能被3整除而不能被2整除，則 $F(n)=F(n-1)+3$ ；
如果 n 既能被2整除又能被3整除，則 $F(n)=F(n-1)+4$ ；
如果 n 既不能被2整除又不能被3整除，則 $F(n)=F(n-1)$ 。
若 $\beta=F(4000)$ ，求 β 的值。

$\beta =$

3. Two circles intersect at B, C as in the figure. If M is the mid-point of BC. $OM = 1$, $OC = 3$, $OE = 5$. If $\gamma = OF$, determine the value of γ.

如圖所示，兩圓相交於 B、C 兩點。M 是 BC 的中點。$OM = 1$, $OC = 3$, $OE = 5$。若 $\gamma = OF$，求 γ 的值。



$\gamma =$

4. If $f(x)=\left(x+\frac{1}{2000}\right)\times\left(x+\frac{1}{2001}\right)\times\cdots\times\left(x+\frac{1}{2019}\right)$ and $\delta = f(1)-1$, determine the value of δ.

若 $f(x)=\left(x+\frac{1}{2000}\right)\times\left(x+\frac{1}{2001}\right)\times\cdots\times\left(x+\frac{1}{2019}\right)$ 以及 $\delta = f(1)-1$，求 δ 的值。

$\delta =$